How theory on parallel robot singularities was used in order to solve sensor-based control problems

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Introduction

• Singularities appearing when observing image features (e.g. with a camera) = a huge challenge in visual servoing
Introduction

• Singularities appearing when observing image features (e.g. with a camera) = a huge challenge in visual servoing

• To the best of our knowledge, only known for three 3-D image points (singularity cylinder)

• Issue with singularities: interaction matrix cannot be inverted anymore = loss of controllability
Introduction

In order to avoid singularities

Increased number of image features (redundancy):
- Pb of local minima
- Proof that there is no singularity?

Determining the singularity cases stays an open problem
Introduction

Recently, the “Hidden Robot Concept” was developed

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea ⇒ Interaction matrix ≡ Inv. Jacobian matrix of a virtual PKM
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- Basic idea $\Rightarrow$ Interaction matrix $\equiv \text{Inv. Jacobian matrix of a virtual PKM}$

For instance, when observing the **leg directions** of the GS platform

- Real robot $= 6$–$U\underline{P}S$
Recently, the “Hidden Robot Concept” was developed

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea $\Rightarrow$ Interaction matrix $\equiv$ Inv. Jacobian matrix of a virtual PKM

For instance, when observing the **leg directions** of the GS platform

- Real robot $= 6-$UPS
- Virtual robot $= 6-$UPS
Introduction

Here

We show how we used the hidden robot concept in order to solve, for the first time, the singularity in

1. the observation of $n$ image points ($n \geq 3$)
2. the observation of three lines
3. the leg-based visual servoing of parallel robots
Observation of an image point

- **Image plane**
- **Camera center**
- **Observation point** $m_1$ with coordinates $(x,y)$
- **Point in 3D space** $M_1$ with coordinates $(X,Y,Z)$
- **Line of sight** $L_1$
Observation of an image point
Observation of an image point

- Camera center $C$
- Image plane
- Point $M_1$ with coordinates $(x,y)$
- Line $L_1$
- Depth $z_1$
Observation of an image point

Image plane

$C$

Camera center

$m_1 (x, y)$

$L_1$

$M_1$
Observation of an image point
Observation of an image point

A **UPS** kinematic chain which allows for the same motion of the point $M_i$
Observation of three image points
Observation of three image points

A 3–UPS robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

\[ \dot{s} = L\tau \quad \text{//} \quad \dot{q} = J_{inv}\tau \]
The three active cardan joints are grouped at the same point.
P3P

The three active cardan joints are grouped at the same point.

Passive prismatic joints

Passive spherical joints
P3P

[Tischler et al., 1998]
P3P

[Tischler et al., 1998]
P3P

Examples of undetermined configurations

[Tischler et al., 1998]
Singularities

Thanks to the hidden robot analogy

Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots

Can be studied by using several (complementary) tools

Singularities

Thanks to the hidden robot analogy
Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots
Can be studied by using several (complementary) tools

In our case (3 points), it can be proven that
The planes $\mathcal{P}_i$ ($i = 1, 2, 3$) and $\mathcal{P}_4$ (containing all 3-D points) have a non-null intersection
Singlarities when observing 3 points
Singularities when observing 3 points
Singularities when observing 3 points
Singularities when observing 3 points

- Cylinder of singularities
- Image plane
- Camera center
- Points and lines

\[ A: \text{Cylinder of singularities} \]

\[ C (\text{camera center}) \]
Singularities when observing $n$ points ($n > 3$)

Possible if and only if

- All singularity cylinders associated with any subset of 3 points have a common intersection
- AND all kernels of the interaction matrices are identical

After (more complex) mathematical derivations, we proved that the conditions of singularity when $n$ coplanar points are observed only appear if and only if all 3-D points and the optical center are located on the same circle.
Singularities when observing $n$ points ($n > 3$)

Examples of undetermined configurations
Simulations
Simulations

\[ \frac{1}{\kappa} \text{ (inverse of the condition number)} \]

Parameter \( s \):

- \( 0 \)
- \( 0.2 \)
- \( 0.4 \)
- \( 0.6 \)
- \( 0.8 \)
- \( 1 \)
- \( 1.2 \)
- \( 1.4 \)

\[ 8 \times 10^{-3} \]
Observation of an image line
Observation of an image line
Observation of an image line
Observation of an image line

\[ \mathcal{L}_i ?? \]
Observation of an image line
Observation of an image line

A $UPRC$ kinematic chain which allows for the same motion of the line $\mathcal{L}_i$. 
Observation of three image lines
Observation of three image lines

A 3–\textit{UPRC} robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

\[
\dot{s} = \mathbf{L}\tau \quad \text{//} \quad \dot{\mathbf{q}} = \mathbf{J}_{\text{inv}}\tau
\]
Thanks to the hidden robot analogy

Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots

Can be studied by using several (complementary) tools

Singularities

Thanks to the hidden robot analogy
Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots
Can be studied by using several (complementary) tools

In our case (3 lines), singu. cond. iff
\[ f_1 = f_{11}^T (f_{21} \times f_{31}) = 0 \]
\[ f_2 = m_{12}^T (m_{22} \times m_{32}) = 0 \]
where \( \xi_{ij} = [f_{ij}^T \ m_{ij}^T]^T \)
Singularities

In order to simplify the problem

- Consider the “zero” platform orientation
- General case obtained by a simple rotation

\[
\begin{bmatrix}
X & Y & Z
\end{bmatrix}^T = \mathcal{R} \begin{bmatrix}
X' & Y' & Z'
\end{bmatrix}^T
\]

(1)

where

- \(X, Y\) and \(Z\): position of the origin of the object frame \(\mathcal{F}_b\) in the camera frame when considering the “zero” platform orientation
- \(X', Y'\) and \(Z'\): position of the origin of the object frame for the considered “non-zero” platform orientation
- \(\mathcal{R}\) the rotation matrix between the two cases
Three coplanar lines with no common intersection point

\[ f_1 = 0 \Leftrightarrow Z = 0 \Rightarrow \text{Lines + optical center in the same plane} \]

\[ f_2 = 0 \Leftrightarrow Z(X^2 + Y^2 - \rho^2) = 0 \Rightarrow \text{Singularity cylinder!} \]
Three coplanar lines with a common intersection point

\[ f_1 = 0 \Rightarrow \text{Singular for any object configuration} \]

\[ f_2 = 0 \Leftrightarrow Z(X^2 + Y^2) = 0 \]

⇒ Camera center \( O \) lies on the line which passes through \( Q \) and which is perpendicular to all vectors \( U_i \)
Three lines in space with a common intersection point

\[ \overrightarrow{OQ} = [X \ Y \ Z]^T, \ U_1 = [1 \ 0 \ 0]^T, \]
\[ U_2 = [a \ b \ 0]^T, \ U_3 = [c \ d \ e]^T \]

(4)

\[ f_1 = 0 \Rightarrow \text{For any object configuration} \]

\[ f_2 = 0 \iff b(adeY^3 + ((-ad^2 + bcd + ae^2)Z + (ac - bd)eX)Y^2 - e(bcX^2 + (ad - bc)Z^2 + 2beXZ)Y + ((-ad^2 + bcd - ae^2)X^2Z + (bd + ac)eXZ^2)) = 0 \]

(5)

\[ \Rightarrow \text{The origin of the body frame belongs to a cubic surface parameterized by } f_2 = 0. \]
Three orthogonal lines in space

\[ (X \ Y \ Z) = (Q \ 1 \ U \ 2 \ U \ 3 \ U \ 1 \ L \ 3 \ L \ 2 \ L) \]

\[ f_1 = 0 \Leftrightarrow aXY + bYZ - cXZ - abc = 0 \]
\[ f_2 = 0 \Leftrightarrow acX - abY + bcZ - XYZ = 0 \]

⇒ Expression \( f_1 \) represents a quadric surface while expression \( f_2 \) is a cubic surface
Three lines, two of them being parallel

\[ f_1 = 0 \iff Z(dZ - eY) = 0 \]
\[ f_2 = 0 \iff Z(X(d^2 + e^2) - cYd - cZe) = 0 \] (7)

- \( Z = 0 \), which occur when the plane \( \mathcal{P} \) containing \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \) also contains the optical center,
- \( eY - dZ = 0 \) is the plane containing \( \mathbf{U}_1, \mathbf{U}_3 \) and the optical center,
- \( X(d^2 + e^2) - cdY - ceZ = 0 \) is the plane containing \( (\mathbf{U}_1 \times \mathbf{U}_3) \), \( \mathbf{U}_3 \) and the optical center.
Three general lines in space

Condition $f_1 = 0$ provides the expression of a quadric surface while $f_2 = 0$ leads to a cubic surface.
Example for three general lines in space

\[ f_1 = 0 \]
\[ f_2 = 0 \]
Simulation 1 (general case)
Simulation 1 (general case)
Simulation 2 (lines are perpendicular)
Simulation 2 (lines are perpendicular)
Leg-based visual servoing of parallel robots

Many approaches, among which

- Direct observation of the end-effector [Paccot et al., 2008]
Leg-based visual servoing of parallel robots

Many approaches, among which

• Leg observation [Özgür et al., 2011]
Leg-based visual servoing of parallel robots

Problems / Questions

• The observation of $m$ leg directions ($m < n$) among the $n$ legs is enough,
Leg-based visual servoing of parallel robots

Problems / Questions

- The observation of $m$ leg directions ($m < n$) among the $n$ legs is enough,
- End-effector convergence issues, even if all leg directions did converge

![Diagram of desired and initial platform configurations with trajectory of the platform.](attachment:figure.png)
Leg-based visual servoing of parallel robots

Problems / Questions

- The observation of $m$ leg directions ($m < n$) among the $n$ legs is enough,
- End-effector convergence issues, even if all leg directions did converge
- Existence of local minima
Leg-based visual servoing of parallel robots

Problems / Questions

• The observation of $m$ leg directions ($m < n$) among the $n$ legs is enough,
• End-effector convergence issues, even if all leg directions did converge
• Existence of local minima
• Interaction model singularities
Leg-based visual servoing of parallel robots

Answers thanks to the hidden robot concept
Leg-based visual servoing of parallel robots

Answers thanks to the hidden robot concept

Idea

We control a virtual robot architecture corresponding to the interaction model (different from the real robot)
Leg-based visual servoing of parallel robots

Idea

We control a virtual robot architecture corresponding to the interaction model (different from the real robot)

Usual encoder-based control

\[ \mathbf{q} \rightarrow \mathbf{x} \ (\mathbf{q}: \text{motor encoder measurements}) \]
Leg-based visual servoing of parallel robots

Idea
We control a virtual robot architecture corresponding to the interaction model (different from the real robot)

Leg-based visual servoing
$u \Rightarrow x$ ($u$: virtual actuator measurements)
Leg-based visual servoing of parallel robots

Leg-observation-based control

Gough-Stewart platform

- Real robot ⇒ 6–UPS
Leg-based visual servoing of parallel robots

Leg-observation-based control

Gough-Stewart platform

- Real robot $\Rightarrow$ 6–UPS
- Hidden (virtual) robot $\Rightarrow$ 3–UPS (case of the minimal observation)
Leg-based visual servoing of parallel robots

Leg-observation-based control

Gough-Stewart platform

- Real robot ⇒ 6–UP_S
- Hidden (virtual) robot ⇒ 3–UP_S (case of the minimal observation)
Leg-based visual servoing of parallel robots

Leg-observation-based control

Gough-Stewart platform

- Real robot $\Rightarrow$ 6–UPS
- Hidden (virtual) robot $\Rightarrow$ 3–UPS (case of the minimal observation)
Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Planar robots: Example of the 3–RRR robot
Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Spatial robots: Example of the Quattro
Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Experimental validation
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 1: Robots which are uncontrollable with the observation of the leg directions

A *PRRRP* robot

Unconstrained translation

Hidden robot: a *PRRRP* robot
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

**Class 2:** Robots which are partially controllable (in their workspace) with the observation of the leg directions
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 3: Robots which are fully controllable (in their workspace) with the observation of the leg directions
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

**Class 4:** Robots which are fully controllable (in their workspace) thanks to additional measurements

A *PRRRP* robot

Hidden robot: a *PRRRP* robot
Conclusions

In this talk,

- I presented a tool named the “hidden robot concept” able to solve the determination of the singularity cases visual servoing based on the observation of geometric features
- we rigorously proved the conditions of singularity for $n$ coplanar points and 3 lines
- we discussed about the generalization of the “hidden robot concept” to other case studies
Conclusions

The hidden robot concept

- a tangible visualization of the mapping between the observation space and the Cartesian space
- allowed to change the way we defined the problem (visual servoing community / mechanical engineering community ⇒ dual problems)
Conclusions

The hidden robot concept

• a tangible visualization of the mapping between the observation space and the Cartesian space
• allowed to change the way we defined the problem (visual servoing community / mechanical engineering community ⇒ dual problems)

Tools used here

• Easily extendable to the rigidity-based control theory
• But useful for you?
Conclusions

Singularity when using bearing measurements
Conclusions

Singularity when using bearing measurements

\[ C \text{ (camera center)} \]

\[ A: \text{Cylinder of singularities} \]

\[ M_1, M_2, M_3 \]

\[ m_1, m_2, m_3 \]
Conclusions

Singularity when using bearing measurements

Uniqueness? ⇒ up to 8 solutions
Adding more measurements? ⇒ Bad choice still leads to singularities
Concluding remarks

Colleagues

Students